NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

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EFFECT OF MISALINEMENT OF STRAIN-GAGE COMPONENTS OF STRAIN ROSETTES

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SUMMARY

A mathematical analysis was made of the effect of misalinement among the components of rectangular and equiangular strain rosettes on the determination of the magnitudes and directions of principal strains. Misalinements of $\pm 2^{\circ}$ among individual gages introduce no serious errors in the computation of principal strains from rosette data. Errors caused by a given misalinement are proportional to the maximum shear strain at the test point and the effect is most important in regions of pure shear.

The equations derived in the analytical investigation provide corrections that are applicable only if the misalinements are known. These equations indicate qualitatively the limits of accuracy with which strain determinations can be made when the possibility of misalinement exists. Verification of the analysis was made by experimental investigation of the behavior of a rosette the misalinement of which was known.

INTRODUCTION

The application of strain rosettes for the determination of magnitude and direction of principal strains at selected points on a stressed surface is well known and several techniques have been developed for analyzing the rosette data into the requisite principal strains and their directions (references 1 to 7). The methods of analysis take for granted that the strain gages constituting the rosette are alined at definite angles to one another; for example, 0°, 45°, and 90° in the rectangular rosette or 0°, 60°, and 120° in the equiangular rosette. Although strain rosettes are now commercially available and there is little reason to doubt the accuracy of their alinement, special applications in stress analysis often require that a rosette be constructed from individual gages. Unless great care is taken to orient the gages of the rosette, some misalinement is generally found to exist among the gages. Speculation arises as to the error that might be introduced by misalinements of 1° or 2° which might normally be expected among the gages of an improvised rosette.

This speculation has a foundation inasmuch as the equations for two-dimensional strain distribution about a strained point indicate that in the region of certain critical directions the linear strain rapidly changes when the direction along which the strain is measured is only slightly varied. Consider, for example, the strain distribution about a point at which the principal strains are 2000 and -1000 microinches per inch. At 43° to the direction of maximum strain, the linear strain is 608 microinches per inch, at 45° the strain is 500 microinches per inch, and at 47° the strain is 392 microinches per inch. Thus, a gage oriented presumably at 45° to the major principal direction but actually at $45 \pm 2^{\circ}$ is capable of a strain indication anywhere between 392 to 608 microinches per inch. This range of strain is, of course, serious; it is conceivable, however, that if the gage is part of a rosette its unfavorable angular orientation may be compensated by the more favorable angular orientations of the other gages along directions in which small increments of angle do not affect the strain indication. The principal strains as computed from the rosette data may therefore not be in error as much as the strain indication of any one gage.

A mathematical analysis of the errors in the determination of direction and magnitude of principal strains that are introduced by misalinement of the gages of a rosette is presented with some deductions that can be drawn from the results of the analysis. A method is also suggested for checking representative rosettes of a given design and construction for exactness of alinement.

EQUATIONS FOR ERRORS DUE TO MISALINEMENT

The mathematical derivations for the effect of misalinement of the strain-gage components of strain rosettes are given in the appendix. The derivations consist of substituting the erroneous strains obtained from the misalined rosette into the formulas generally used for reducing data obtained from properly alined rosettes. The apparent principal strains and their directions as they would be calculated by an operator not knowing that the gages of the rosette were misalined are compared with the true principal strains and their directions and the errors introduced by misalinement thereby determined in the form of correction factors. The errors are determined for rosettes of rectangular and equiangular design and are defined by the following equations:

Case I - Rectangular rosette, gages theoretically 0°, 45°, and 90° apart. -

$$\epsilon_p' = \epsilon_p + 0.00875\ \gamma_{max}\ M \qquad (1)$$

where M is

$$[(\Delta_3 - \Delta_1) \sin 2\theta_1 + (\Delta_2 - 0.5\Delta_1 - 0.5\Delta_3) \sin 4\theta_1 + (\Delta_3 - \Delta_1) \sin^2 2\theta_1]$$

$$\epsilon_q' = \epsilon_q + 0.00875\ \gamma_{max}\ M \qquad (2)$$

where M is

$$[(\Delta_3 - \Delta_1) \sin 2\theta_1 - (\Delta_2 - 0.5\Delta_1 - 0.5\Delta_3) \sin 4\theta_1 - (\Delta_3 - \Delta_1) \sin^2 2\theta_1]$$

$$\theta_p' = \theta_p - [\ \Delta_2 \cos^2 2\theta_1 + 0.5\ (\Delta_1 + \Delta_3) \sin^2 2\theta_1 - 0.25\ (\Delta_1 - \Delta_3) \sin 4\theta_1] \qquad (3)$$

and

ϵ_p, ϵ_q true major and minor principal strains at test point

ϵ_p', ϵ_q' apparent major and minor principal strains as calculated by neglecting misalinement of gage components of rosette

γ_{max} maximum shear strain at test point

θ_1 theoretical angle of reference gage 1 of rosettes measured counterclockwise from direction of ϵ_p. (The gages of a rosette are arbitrarily numbered 1, 2, and 3. In rectangular rosettes, gages 2 and 3 are theoretically oriented at 45° and 90° counterclockwise to gage 1; in equiangular rosettes gages 2 and 3 are oriented at 60° and 120° counterclockwise to gage 1.)

θ_p, θ_p' directions of ϵ_p and ϵ_p' measured counterclockwise from gage 1, degrees

Δ_1,Δ_2,Δ_3 misalinement of gages 1, 2, and 3. (Misalinement is defined as the angle in degrees between the theoretical and the actual directions of gages 1, 2, and 3. Δ_1, Δ_2, and Δ_3 are positive if gages 1, 2, and 3 are oriented at slight counterclockwise angles to their theoretical directions as shown in fig. 2.)

Case II - Equiangular rosette, gages theoretically 0°, 60°, and 120° apart. -

$$\epsilon_p' = \epsilon_p + 0.00875\ \gamma_{max}\ M \tag{4}$$

where M is

$$\left[\frac{\sqrt{3}}{3}(\Delta_3 - \Delta_2)\ (\cos 2\theta_1 - \cos 4\theta_1) + \frac{1}{3}(\Delta_2 + \Delta_3 - 2\Delta_1)(\sin 2\theta_1 + \sin 4\theta_1)\right]$$

$$\epsilon_q' = \epsilon_q + 0.00875\ \gamma_{max}\ M \tag{5}$$

where M is

$$\left[\frac{\sqrt{3}}{3}(\Delta_3 - \Delta_2)(\cos 2\theta_1 + \cos 4\theta_1) + \frac{1}{3}(\Delta_2 + \Delta_3 - 2\Delta_1)(\sin 2\theta_1 - \sin 4\theta_1)\right]$$

$$\theta_p' = \theta_p - M \tag{6}$$

where M is

$$\left[\frac{\sqrt{3}}{6}(\Delta_3 - \Delta_2)\ \sin 4\theta_1 + \frac{1}{6}(\Delta_2 + \Delta_3)(2\ \cos^2 2\theta_1 + 1) + \frac{2}{3}\Delta_1\ \sin^2 2\theta_1\right]$$

EFFECT OF MISALINEMENT

The user of a rosette does not, in general, know how much misalinement is present among the gages constituting the rosette; hence, the proper corrections as defined by equations (1) to (6) cannot be applied. The equations have utility, however, in pointing to qualitative errors that might be expected in a given application and to some general principles that constitute good technique in interpreting strain-rosette data. Several important deductions that may be made from the equations are summarized as follows:

1. The error in the determination of magnitude and directions of principal strains resulting from the use of a misalined rosette depends upon the orientation of the reference gage of the rosette relative to the principal axes as well as misalinements among the gages within the rosette. This factor is to be expected inasmuch as the general orientation of the rosette determines whether one or more gages of the rosette will be oriented along a critical direction in which the strain rapidly varies with slight changes in angular orientation.

2. Small angular misalinements of the strain gages of a rosette can result in determinations of principal strains that are only moderately different from the true principal strains. Unless determinations accurate within better than ±2 percent of principal strains are required, misalinements of 1° or 2° may be tolerated.

An inspection of equations (1) to (6) shows that, assuming all the Δ's to be of the same magnitude $\underline{\Delta}$ (but of unknown sign), the limits of the possible errors are given by:

$$\left.\begin{aligned} \epsilon_p' - \epsilon_p &< 0.052\ \gamma_{max}\ \underline{\Delta} \\ \epsilon_q' - \epsilon_q &< 0.052\ \gamma_{max}\ \underline{\Delta} \\ \theta_p' - \theta_p &< 2.5\ \Delta \end{aligned}\right\}\ \text{rectangular rosette}$$

$$\left.\begin{aligned} \epsilon_p' - \epsilon_p &< 0.032\ \gamma_{max}\ \underline{\Delta} \\ \epsilon_q' - \epsilon_q &< 0.032\ \gamma_{max}\ \underline{\Delta} \\ \theta_p' - \theta_p &< 1.6\ \Delta \end{aligned}\right\}\ \text{equiangular rosette}$$

3. When the problem of misalinement is considered in a practical application, there appears to be little inherent advantage in using either the rectangular or equiangular rosette in preference to the other. Although the equiangular design may be better than the rectangular for a given misalinement and reference-gage orientation, random misalinements and reference-gage orientations may produce as much error in one type of rosette as in the other.

4. For a given misalinement among the gages of a rosette and orientation of the reference gage relative to the principal axes, the error in the determination of the principal strains is proportional to the maximum shear strain at the test point. Thus, particular concern should be given to alinement of gages when measurements are to be made in fields of pure shear.

5. The error in the determination of principal directions from data obtained from misalined gages of a strain rosette is of the same order of magnitude as the misalinements of the gages within the rosette; that is, no disproportionate errors in locating the principal directions are introduced by moderate misalinements in the gages of the rosette.

6. As a deduction from deduction 5 and the property of rectangular rosettes given in deduction 3, it follows that an accurate method of determining the principal strains at a desired point, even

when some doubt exists as to the proper alinement of gages in available rosettes, is to determine first the approximate directions of principal strains by means of a rosette and then to determine the magnitude of the principal strains from a second rectangular rosette so oriented that the reference gage lies very nearly along a principal direction.

EXPERIMENTAL CHECK ON MISALINEMENT OF ROSETTE GAGES

An experimental survey was conducted to determine the errors introduced by misalinement of the strain gages of a rectangular rosette. The rosette was constructed of three bobbin-wound Bakelite-impregnated strain gages of 1/16-inch gage length. Gages of 1/16-inch effective length were used in preference to larger sizes in order to obtain a rosette of very small dimensions. These small gages have been used at the Cleveland laboratory for some time and have proved to be as reliable as any of the larger sizes. Because no special precautions were taken to aline the gages perfectly, some misalinement was expected among the gages. The rosette was mounted at the center of an accurately machined 16-sided duralumin polygon similar to that used by Dow in reference 8. Figure 1 shows the polygon and the attached gages.

The polygon was then mounted in a hydraulic testing machine of 120,000-pound capacity and the strain indication of each of the three gages was observed as the load on the block was increased in increments of 20,000 pounds. The block was then rotated $22\frac{1}{2}^{\circ}$ and the strain indication of each of the gages again observed for equal increments of load. Surveys were conducted for each $22\frac{1}{2}^{\circ}$ rotation for 180°.

The exact alinement of the gages relative to one another was then determined in the following manner:

If three successive strain indications are obtained from a single gage of the rosette as the duralumin block is loaded on pairs of faces 45° apart, these indications can be considered as the indications from the three gages of a perfectly alined rosette. Because the duralumin block is accurately machined, the faces are exactly 45° apart; hence, the gages of the hypothetical rosette relative to the principal axes of strain can then be determined from the three strain readings and, inasmuch as the principal axes of the loaded polygon are known to be perpendicular and parallel, respectively, to the loaded faces, the direction of each gage can be determined in relation to known directions on the duralumin block. In this manner it was determined that the rosette tested consisted of gages at angles of 0.875°, 48°, and 92.08° relative to the assumed direction of gage 1 instead of the ideal 0°, 45°, and 90°; hence,

$$\Delta_1 = 0.875^\circ,\ \Delta_2 = 3^\circ,\ \text{and}\ \Delta_3 = 2.08^\circ$$

A comparison of the true principal strains, the principal strains as determined from simultaneous readings of the rosette, and the theoretical principal strains as calculated from the equations derived in this report is given in table I. Experimental values closely agree with the values predicted by the equations derived in this report. Because the experimental results do not excessively deviate from the theoretical results, misalinements of the order of 2° can be tolerated with only moderate errors in computed apparent principal strains.

The strain at an angle θ to the major principal strain is given by the formula

$$\epsilon_\theta = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q)\cos 2\theta\right] \tag{7}$$

A perfect strain gage oriented along the direction θ will yield an indication proportional to the strain as given in equation (7). Short gages, such as those used in this experiment, are affected by cross strains as well as by the strain along which the gage is alined but the strain indication of the gage can still be given by the relation

$$\epsilon_\theta = \alpha + \beta \cos 2\theta \tag{8}$$

where α and β are variables that depend on the principal strains as well as on the size of the gages. When the data from a rosette consisting of gages that are subject to cross strains are substituted in the usual formulas for determining principal strains, the calculated strains are not the principal strains but the strains numerically equal to $(\alpha + \beta)$ and $(\alpha - \beta)$.

Because the derivations of the correction factors did not take into account cross strains, these corrections apply to the principal strains only when the gages employed are not subject to cross strains and to the values of $(\alpha + \beta)$ and $(\alpha - \beta)$ when the gages are subjected to cross strains. This experiment verified that the corrections apply to the values of $(\alpha + \beta)$ and $(\alpha - \beta)$.

CONCLUDING REMARKS

The method used to determine the misalinements present in the rosette tested also provides a general method of checking representative rosettes of a given design for accuracy of alinement. A sample rosette may be applied to the center of an accurately machined polygon and the orientation of each gage independently determined relative to fixed axes on the polygon. The relative orientation of the gages among each other may thereby be very accurately determined.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, April 17, 1946.

APPENDIX - DERIVATION OF EQUATIONS

The true strain ϵ_θ at an angle θ to the direction of the major principal axis is given by the following equation from reference 5 (p. 30):

$$\epsilon_\theta = \frac{(\epsilon_p + \epsilon_q)}{2} + \frac{(\epsilon_p - \epsilon_q)}{2} \cos 2\theta \qquad (9)$$

Gage 1 is at an angle of $(360^\circ + \Delta_1)$ to its assumed direction and therefore at an angle of $(\theta_1 + 360^\circ + \Delta_1)$ to the major principal axis. Hence

$$\epsilon_1' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q) \cos 2(\theta_1 + 360 + \Delta_1)\right]$$

$$\epsilon_1' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q)(\cos 2\theta_1 \cos 2\Delta_1 - \sin 2\theta_1 \sin 2\Delta_1)\right]$$

where

ϵ_1', ϵ_2', ϵ_3' strains indicated by gages 1, 2, and 3 of rosette

θ_1 angle of gage 1 from direction of major principal axis, deg

If Δ_2 is a small angle expressed in radians

$$\cos 2\Delta_2 = 1$$

$$\sin 2\Delta_2 = 2\Delta_2$$

Or if Δ_2 is expressed in degrees, as is done in this report

$$\cos 2\Delta_2 = 1$$

$$\sin 2\Delta_2 = 0.035 \Delta_2$$

and

$$\epsilon_1' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q)(\cos 2\theta_1 - 0.035 \Delta_1 \sin 2\theta_1)\right] \qquad (10)$$

Case I - rectangular rosette. - Gage 2 is at an angle of $(45^\circ + \Delta_2)$ to gage 1 and therefore at an angle of $(\theta_1 + 45^\circ + \Delta_2)$ to the major principal axis. The strain ϵ_2' is expressed by

$$\epsilon_2' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q) \cos 2(\theta_1 + 45 + \Delta_2)\right]$$

which reduces to

$$\epsilon_2' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) - (\epsilon_p - \epsilon_q)(\sin 2\theta_1 + 0.035\ \Delta_2 \cos 2\theta_1)\right] \qquad (11)$$

Gage 3 is at an angle of $(90^\circ + \Delta_3)$ to gage 1 and therefore at an angle of $(\theta_1 + 90^\circ + \Delta_3)$ to the major principal axis. The strain ϵ_3' is expressed by

$$\epsilon_3' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q) \cos 2(\theta_1 + 90 + \Delta_3)\right]$$

which reduces to

$$\epsilon_3' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) - (\epsilon_p - \epsilon_q)(\cos 2\theta_1 - 0.035\ \Delta_3 \sin 2\theta_1)\right] \qquad (12)$$

If the values from equations (10), (11), and (12) are substituted in the usual equations for calculating principal strains,

$$\epsilon_{p,q} = \frac{\epsilon_1 + \epsilon_3}{2} \pm \frac{\sqrt{2}}{2} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2} \qquad (13)$$

Then apparent principal strains ϵ_p' and ϵ_q' are calculated

$$\epsilon_p' = \epsilon_p + 0.00875\ \gamma_{max}\ M \qquad (1)$$

where M is

$$\left[(\Delta_3 - \Delta_1) \sin 2\theta_1 + (\Delta_2 - 0.5\Delta_1 - 0.5\Delta_3) \sin 4\theta_1 + (\Delta_3 - \Delta_1) \sin^2 2\theta_1\right]$$

$$\epsilon_q' = \epsilon_q + 0.00875\ \gamma_{max}\ M \qquad (2)$$

where M is

$$\left[(\Delta_3 - \Delta_1) \sin 2\theta_1 - (\Delta_2 - 0.5\Delta_1 - 0.5\Delta_3) \sin 4\theta_1 - (\Delta_3 - \Delta_1) \sin^2 2\theta_1\right]$$

If the values from equations (10), (11), and (12) are substituted in the usual equation for calculating the direction of the major principal axis measured from gage 1 of the rosette,

$$\tan 2\theta_p = \frac{2\epsilon_2 - \epsilon_1 - \epsilon_3}{\epsilon_1 - \epsilon_3} \qquad (14)$$

Then

$$\tan 2\theta_p' = - M \tag{15}$$

where M is

$$\frac{2 \sin 2\theta_1 - 0.035\Delta_1 \sin 2\theta_1 + 0.035\Delta_3 \sin 2\theta_1 + 0.07\Delta_2 \cos 2\theta_1}{- 0.035\Delta_1 \sin 2\theta_1 - 0.035\Delta_3 \sin 2\theta_1 + 2 \cos 2\theta_1}$$

Equation (15) defines an apparent angle θ_p', which would be calculated as the direction of the major principal axis measured counterclockwise from the direction of ϵ_1. Let Δ be the displacement of the apparent principal direction from the true principal direction, then by figure 2

$$\theta_p' = 360 + \Delta - \theta_1$$

Therefore

$$\tan 2\theta_p' = \tan 2\,(360 + \Delta - \theta_1) = -\tan\,(2\theta_1 - 2\Delta)$$

$$= -\frac{\tan 2\theta_1 - \tan 2\Delta}{1 + \tan 2\theta_1 \tan 2\Delta} = -\frac{\tan 2\theta_1 - 0.035\Delta}{1 + 0.035\Delta \tan 2\theta_1}$$

$$\tan 2\theta_p' = -\frac{\sin 2\theta_1 - 0.035\Delta \cos 2\theta_1}{\cos 2\theta_1 + 0.035\Delta \sin 2\theta_1} \tag{16}$$

After the two expressions for $\tan 2\theta_p'$ in equations (15) and (16) have been equated, the value of Δ may be calculated.

$$\Delta = -[\Delta_2 \cos^2 2\theta_1 + 0.5\,(\Delta_1 + \Delta_3) \sin^2 2\theta_1 - 0.25\,(\Delta_1 - \Delta_3) \sin 4\theta_1] \tag{17}$$

Therefore

$$\theta_p' = \theta_p + \Delta = \theta_p - M \tag{3}$$

where M is

$$[\Delta_2 \cos^2 2\theta_1 + 0.5(\Delta_1 + \Delta_3) \sin^2 2\theta_1 - 0.25\,(\Delta_1 - \Delta_3) \sin 4\theta_1]$$

Case II - equiangular rosette. - The expression for ϵ_1' of the equiangular rosette is the same as that for ϵ_1' of the rectangular rosette.

$$\epsilon_1' = \frac{1}{2}\,[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q) \cos 2(\theta_1 + 360 + \Delta_1)]$$

$$= \frac{1}{2}\,[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q)(\cos 2\theta_1 - 0.035\,\Delta_1 \sin 2\theta_1)] \tag{10}$$

$$\epsilon_2' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q)\cos 2(\theta_1 + \Delta_2 + 60)\right]$$

$$= \frac{1}{2} M \tag{18}$$

where M is

$$\left\{(\epsilon_p + \epsilon_q) - (\epsilon_p - \epsilon_q)\left[\left(\frac{1}{2} + 0.0175\sqrt{3}\Delta_2\right)\cos 2\theta_1 + \left(\frac{\sqrt{3}}{2} - 0.0175\Delta_2\right)\sin 2\theta_1\right]\right\}$$

$$\epsilon_3' = \frac{1}{2}\left[(\epsilon_p + \epsilon_q) + (\epsilon_p - \epsilon_q)\cos 2(\theta_1 + \Delta_3 + 120)\right]$$

$$= \frac{1}{2} M \tag{19}$$

where M is

$$\left\{(\epsilon_p + \epsilon_q) - (\epsilon_p - \epsilon_q)\left[\left(\frac{1}{2} - 0.0175\sqrt{3}\Delta_3\right)\cos 2\theta_1 - \left(\frac{\sqrt{3}}{2} + 0.0175\Delta_3\right)\sin 2\theta_1\right]\right\}$$

If equations (10), (18), and (19) are substituted in the expressions for principal strains

$$\epsilon_{p,q} = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3} \pm \frac{\sqrt{2}}{3}\sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_1 - \epsilon_3)^2 + (\epsilon_2 - \epsilon_3)^2} \tag{20}$$

When small quantities of the second order of magnitude are neglected

$$\epsilon_p' = \epsilon_p + 0.00875\,\gamma_{max}\,M \tag{4}$$

where M is

$$\left[\frac{\sqrt{3}}{3}(\Delta_3 - \Delta_2)(\cos 2\theta_1 - \cos 4\theta_1) + \frac{1}{3}(\Delta_2 + \Delta_3 - 2\Delta_1)(\sin 2\theta_1 + \sin 4\theta_1)\right]$$

$$\epsilon_q' = \epsilon_q + 0.00875\,\gamma_{max}\,M \tag{5}$$

where M is

$$\left[\frac{\sqrt{3}}{3}(\Delta_3 - \Delta_2)(\cos 2\theta_1 + \cos 4\theta_1) + \frac{1}{3}(\Delta_2 + \Delta_3 - 2\Delta_1)(\sin 2\theta_1 - \sin 4\theta_1)\right]$$

The angle of the major principal axis measured counterclockwise from gage 1 is given by

$$\tan 2\theta_p = \frac{\sqrt{3}\ (\epsilon_2 - \epsilon_3)}{2\ \epsilon_1 - \epsilon_2 - \epsilon_3} \tag{21}$$

With equations (10), (18), and (19) substituted

$$\tan 2\theta_p{}' = -\ M \tag{22}$$

where M is

$$\frac{3(0.0175)(\Delta_2 + \Delta_3)\ \cos 2\theta_1 + \left[3 + \sqrt{3}\ (0.0175)(\Delta_3 - \Delta_2)\right] \sin 2\theta_1}{\left[3 + 0.0175\sqrt{3}\ (\Delta_2 - \Delta_3)\right] \cos 2\theta_1 - \left[0.07\Delta_1 + 0.0175\ (\Delta_2 + \Delta_3)\right] \sin 2\theta_1}$$

With equation (22) equated to equation (16) for an equiangular rosette Δ becomes

$$\Delta = -\left[\frac{\sqrt{3}}{6}\ (\Delta_3 - \Delta_2)\ \sin 4\theta_1 + \frac{1}{6}\ (\Delta_2 + \Delta_3)(2\ \cos^2 2\theta_1 + 1) + \frac{2}{3}\ \Delta_1 \sin^2 2\theta_1\right] \tag{23}$$

Hence

$$\theta_p{}' = \theta_p - M \tag{6}$$

where M is

$$\left[\frac{\sqrt{3}}{6}\ (\Delta_3 - \Delta_2)\ \sin 4\theta_1 + \frac{1}{6}\ (\Delta_2 + \Delta_3)\ (2\ \cos^2 2\theta_1 + 1) + \frac{2}{3}\ \Delta_1\ \sin^2 2\theta_1\right]$$

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TABLE I - EXPERIMENTAL AND PREDICTED ERRORS CAUSED BY MISALINEMENT

OF STRAIN GAGES IN A RECTANGULAR ROSETTE

[All strains are in microin./in.]

Theoretical angle of gage 1 with major principal strain, θ_1 (deg)	Major apparent principal strain			Minor apparent principal strain			Major principal axis orientation to gage 1 (deg)		
	True	Experimental	Predicted by equation	True	Experimental	Predicted by equation	True	Experimental	Predicted by equation
0	175	177	175	-283	-285	-283	-0.9	-2.2	-3.0
$22\frac{1}{2}$	175	186	187	-283	-286	-288	-23.9	-24.1	-25.0
45	175	188	185	-283	-284	-283	-45.9	-45.5	-46.5
$67\frac{1}{2}$	175	178	175	-283	-278	-276	-68.4	-68.6	-69.5
90	175	178	175	-283	-284	-283	-90.9	-92.0	-93.0
$112\frac{1}{2}$	175	179	180	-283	-291	-295	-113.4	-114.7	-115.0
135	175	175	175	-283	-291	-292	-135.9	-135.6	-136.5
$157\frac{1}{2}$	175	169	168	-283	-283	-283	-158.4	-158.7	-159.5
180	175	177	175	-283	-285	-283	-.9	-2.2	-3.0

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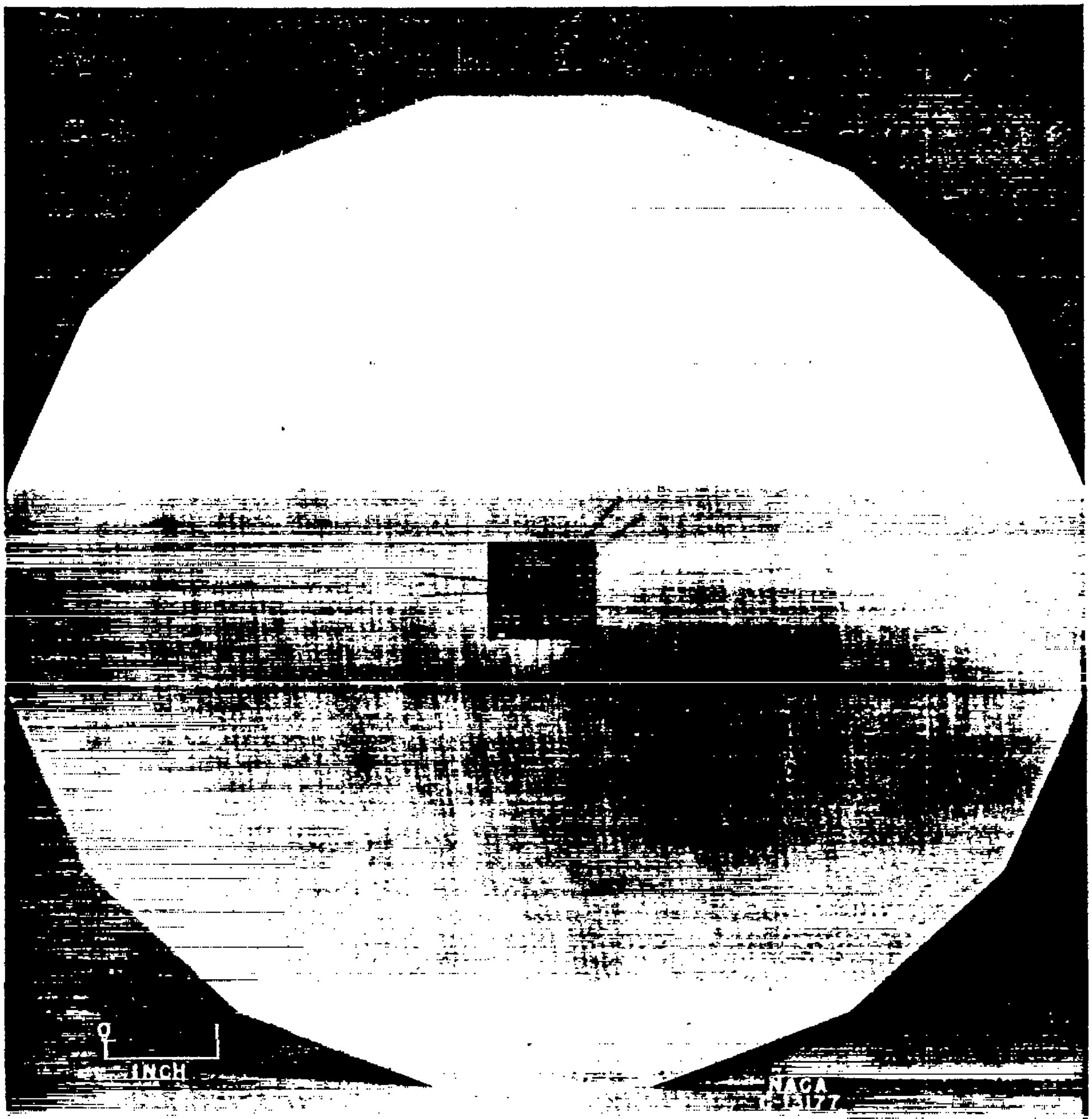
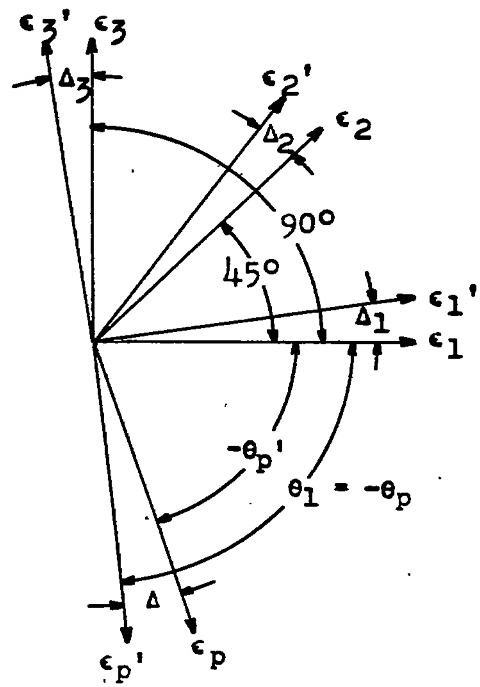
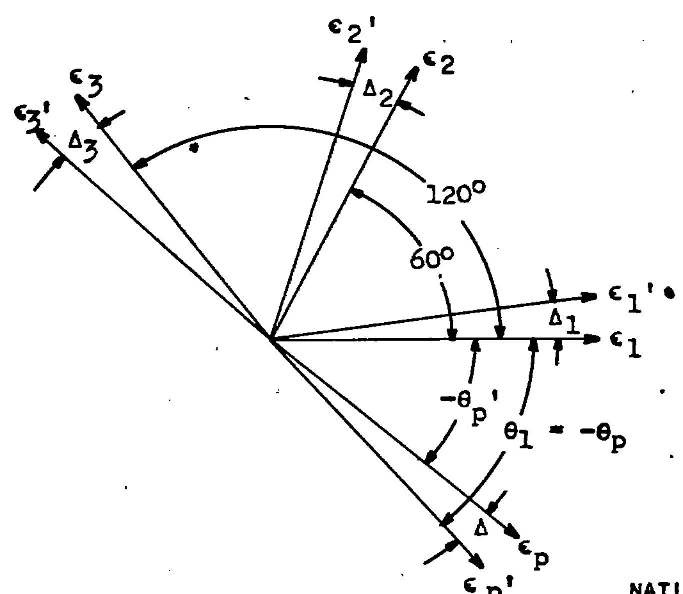


Figure 1. - Polygon for testing alinement of strain gages in rosettes. Three strain gages one-sixteenth by one-sixteenth inch shown mounted in form of rectangular rosette.



(a) Rectangular rosette.



(b) Equiangular rosette.

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Figure 2. - Rosette strain vectors.